

DATA STRUCTURES AND ALGORITHMS

LECTURE 27

KRUSKAL'S ALGORITHM

IMRAN IHSAN
ASSISTANT PROFESSOR
AIR UNIVERSITY, ISLAMABAD

KRUSKAL'S ALGORITHM

- Kruskal's algorithm sorts the edges by weight and goes through the edges from least weight to greatest weight adding the edges to an empty graph so long as the addition does not create a cycle
- The halting point is:
 - When $|V| - 1$ edges have been added
 - In this case we have a minimum spanning tree
 - We have gone through all edges, in which case, we have a forest of minimum spanning trees on all connected sub-graphs



EXAMPLE

- Consider the game of Risk from Parker Brothers
 - A game of world domination
 - The world is divided into 42 connected regions



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EXAMPLE

- Consider the game of Risk from Parker Brothers
 - A game of world domination
 - The world is divided into 42 connected regions
 - The regions are vertices and edges indicate adjacent regions



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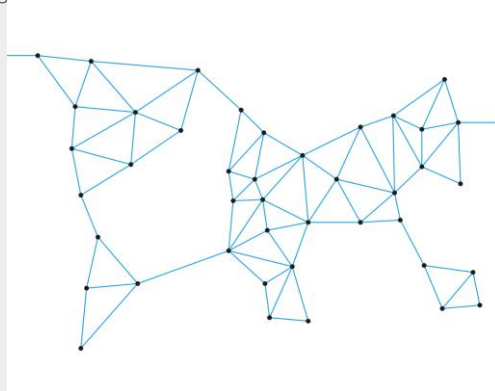
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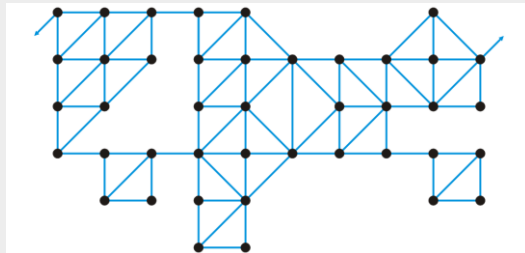
EXAMPLE

- Consider the game of Risk from Parker Brothers
 - A game of world domination
 - The world is divided into 42 connected regions
 - The regions are vertices and edges indicate adjacent regions
 - The graph is sufficient to describe the game



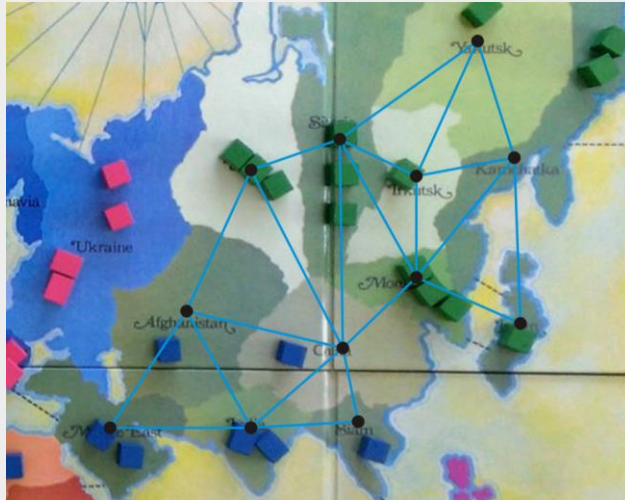
EXAMPLE

- Consider the game of Risk from Parker Brothers
 - Here is a more abstract representation of the game board
 - Suddenly, it's less interesting: "I've conquered the graph!"



EXAMPLE

- We'll focus on Asia



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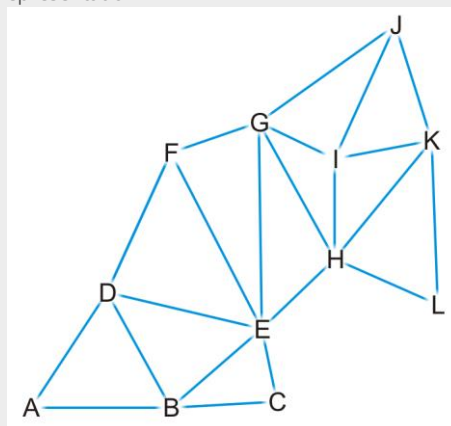
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EXAMPLE

- Here is our abstract representation



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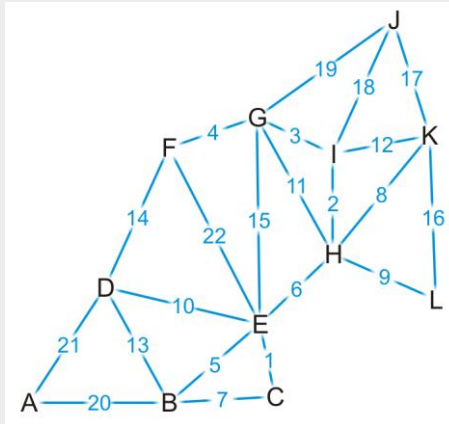
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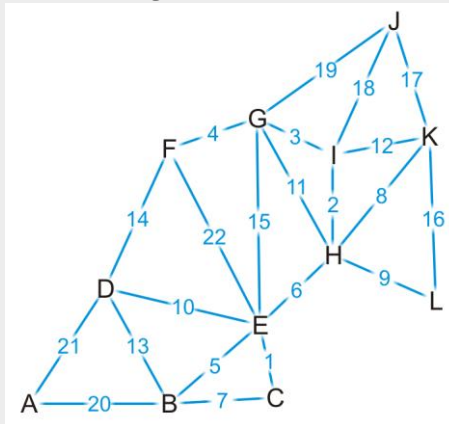
EXAMPLE

- Let us give a weight to each of the edges



EXAMPLE

- First, we sort the edges based on weight

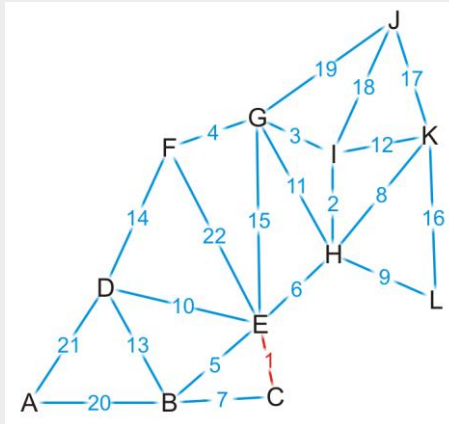


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We start by adding edge {C, E}

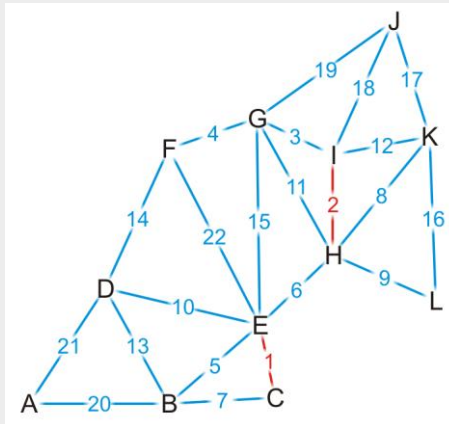


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We add edge {H, I}

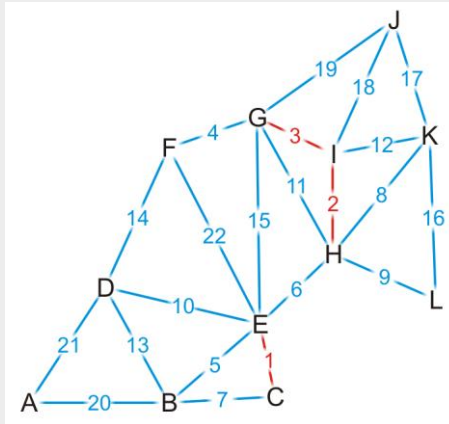


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
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- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We add edge {G, I}

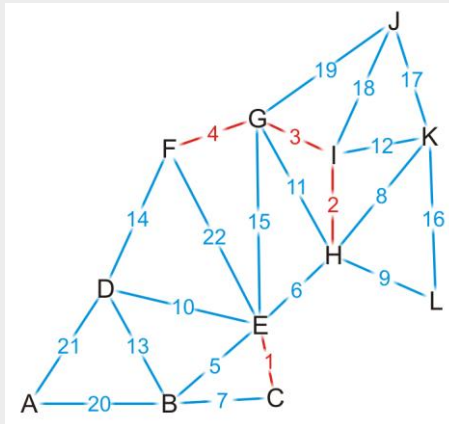


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We add edge {F, G}

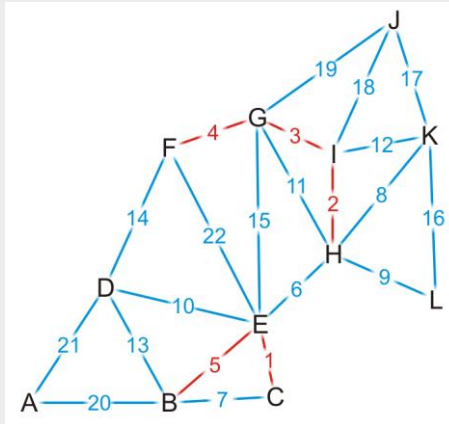


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We add edge {B, E}

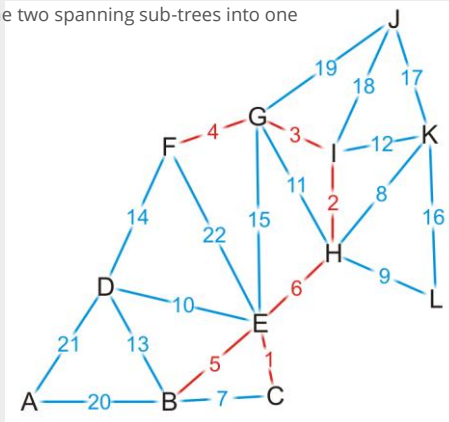


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We add edge {E, H}
 - This coalesces the two spanning sub-trees into one

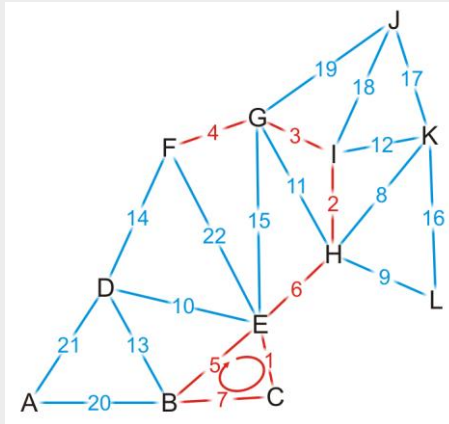


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We try adding {B, C}, but it creates a cycle



- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



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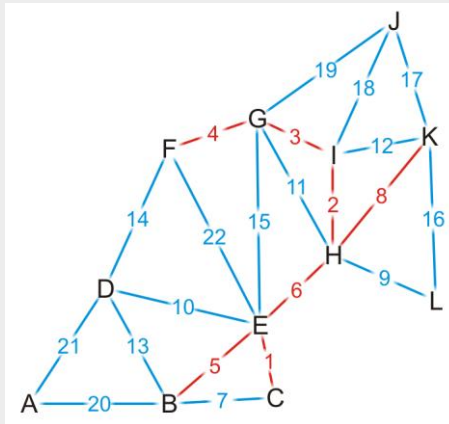
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EXAMPLE

- We add edge {H, K}



- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



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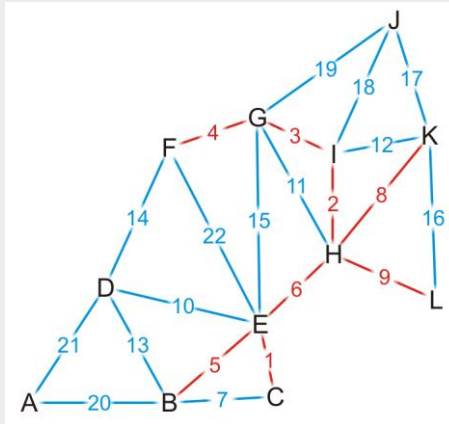
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EXAMPLE

- We add edge {H, L}

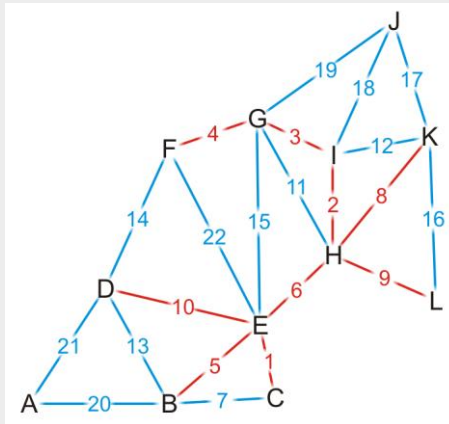


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We add edge {D, E}

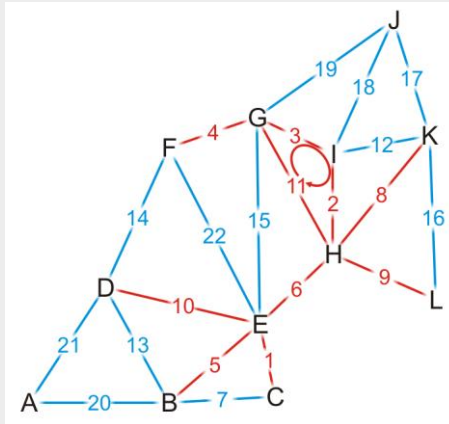


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We try adding {G, H}, but it creates a cycle

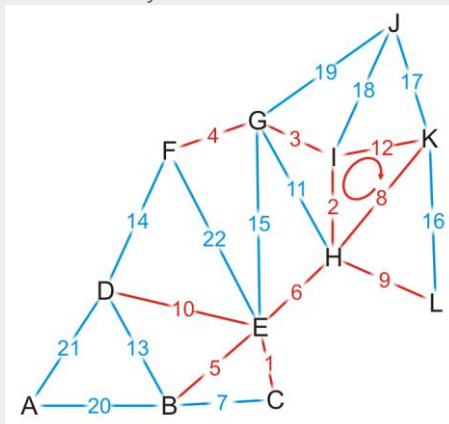


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We try adding {I, K}, but it creates a cycle

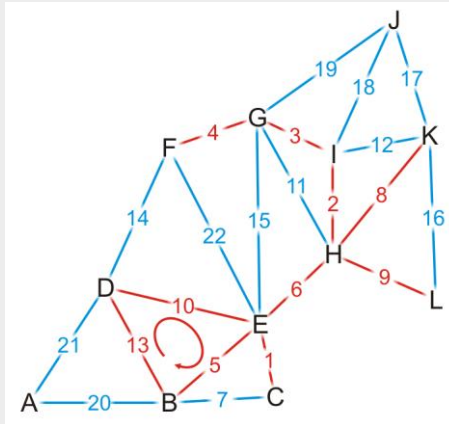


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We try adding {B, D}, but it creates a cycle

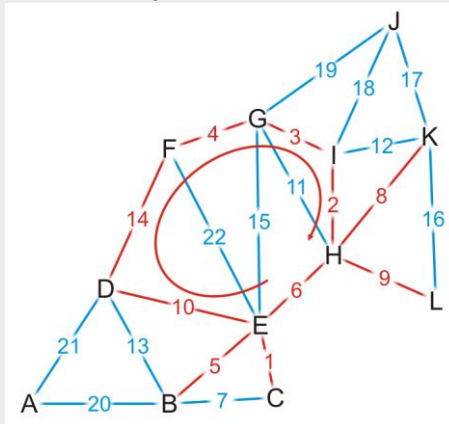


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We try adding {D, F}, but it creates a cycle

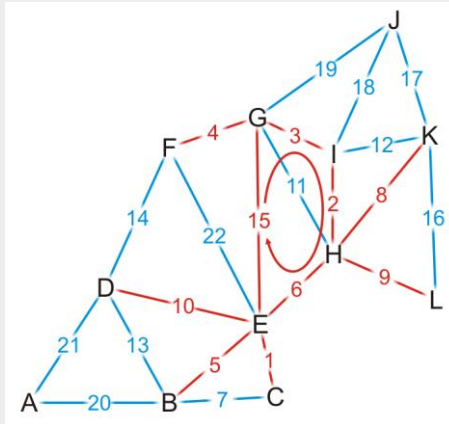


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- We try adding {E, G}, but it creates a cycle

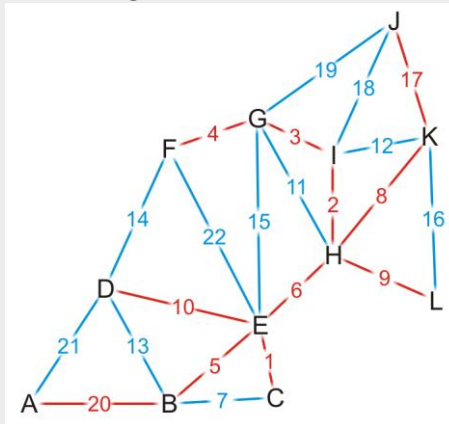


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- By observation, we can still add edges {J, K} and {A, B}

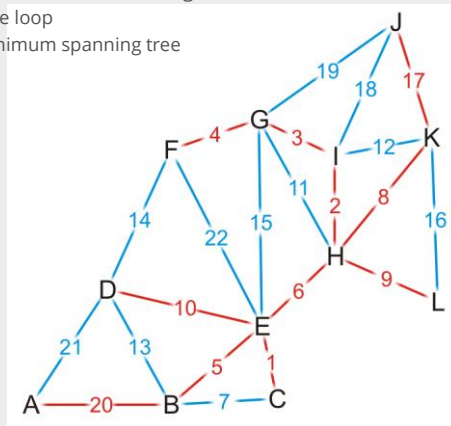


- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}



EXAMPLE

- Having added {A, B}, we now have 11 edges
 - We terminate the loop
 - We have our minimum spanning tree



- {C, E}
- {H, I}
- {G, I}
- {F, G}
- {B, E}
- {E, H}
- {B, C}
- {H, K}
- {H, L}
- {D, E}
- {G, H}
- {I, K}
- {B, D}
- {D, F}
- {E, G}
- {K, L}
- {J, K}
- {J, I}
- {J, G}
- {A, B}
- {A, D}
- {E, F}

