

DATA STRUCTURES AND ALGORITHMS

LECTURE 24

IDENTIFYING BIPARTITE GRAPH

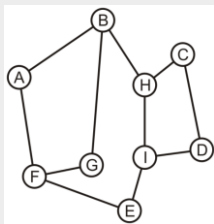
IMRAN IHSAN
ASSISTANT PROFESSOR
AIR UNIVERSITY, ISLAMABAD

BIPARTITE GRAPHS

DEFINITION

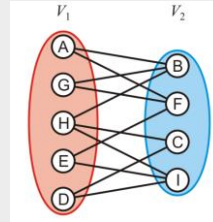
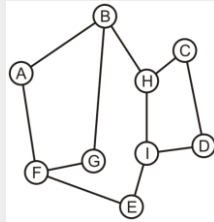
A *bipartite graph* is a graph where the vertices V can be divided into two disjoint sets V_1 and V_2 such that **every** edge has one vertex in V_1 and the other in V_2

Consider this graph: is it bipartite?



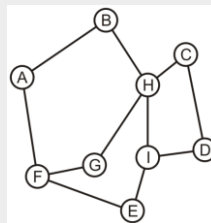
BIPARTITE GRAPHS

- Yes:
 - With a little work, it is possible to determine that we can decompose the vertices into two disjoint sets



BIPARTITE GRAPHS

Is this graph bipartite?



In this case, it is not a bipartite graph

- Can we find a traversal that will determine if a graph is bipartite?



BIPARTITE GRAPHS

Consider using a breadth-first traversal for a connected graph:

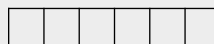
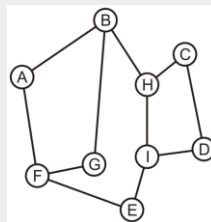
- Choose a vertex, mark it belonging to V_1 and push it onto a queue
- While the queue is not empty, pop the front vertex v and
 - Any adjacent vertices that are already marked must belong to the set not containing v , otherwise, the graph is not bipartite (we are done); while
 - Any unmarked adjacent vertices are marked as belonging to the other set and they are pushed onto the queue
- If the queue is empty, the graph is bipartite



BIPARTITE GRAPHS

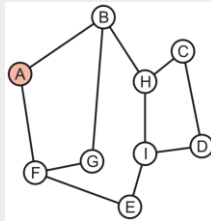
With the first graph, we can start with any vertex

- We will use colours to distinguish the two sets



BIPARTITE GRAPHS

Push A onto the queue and colour it red



www.au.edu.pk

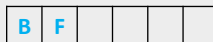
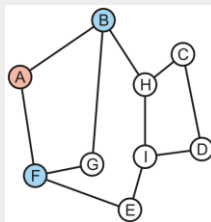
Air University, Islamabad

www.imranhsan.com



BIPARTITE GRAPHS

Pop A and its two neighbours are not marked:
• Mark them as blue and push them onto the queue



www.au.edu.pk

Air University, Islamabad

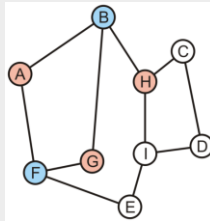
www.imranhsan.com



BIPARTITE GRAPHS

Pop B—it is blue:

- Its one marked neighbour, A, is red
- Its other neighbours G and H are not marked: mark them red and push them onto the queue



www.au.edu.pk

Air University, Islamabad

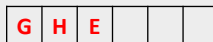
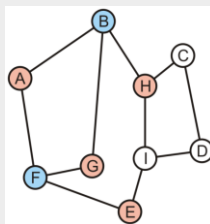
www.imranhsan.com



BIPARTITE GRAPHS

Pop F—it is blue:

- Its two marked neighbours, A and G, are red
- Its neighbour E is not marked: mark it red and pus it onto the queue



www.au.edu.pk

Air University, Islamabad

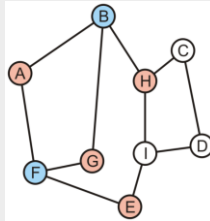
www.imranhsan.com



BIPARTITE GRAPHS

Pop G—it is red:

- Its two marked neighbours, B and F, are blue



www.au.edu.pk

Air University, Islamabad

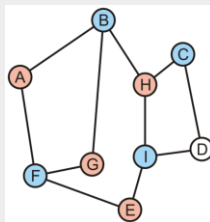
www.imranhsan.com



BIPARTITE GRAPHS

Pop H—it is red:

- Its marked neighbour, B, is blue
- It has two unmarked neighbours, C and I; mark them blue and push them onto the queue



www.au.edu.pk

Air University, Islamabad

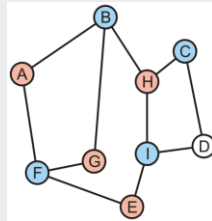
www.imranhsan.com



BIPARTITE GRAPHS

Pop E—it is red:

- Its marked neighbours, F and I, are blue



www.au.edu.pk

Air University, Islamabad

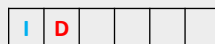
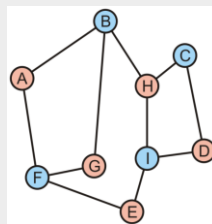
www.imranhsan.com



BIPARTITE GRAPHS

Pop C—it is blue:

- Its marked neighbour, H, is red
- Mark D as red and push it onto the queue



www.au.edu.pk

Air University, Islamabad

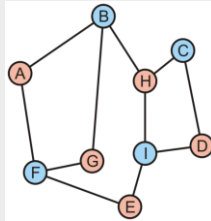
www.imranhsan.com



BIPARTITE GRAPHS

Pop I—it is blue:

- Its marked neighbours, H, D and E, are all red



D					
---	--	--	--	--	--



www.au.edu.pk

Air University, Islamabad

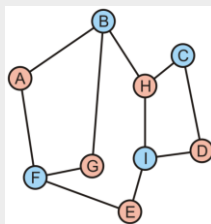
www.imranhsan.com



BIPARTITE GRAPHS

Pop D—it is red:

- Its marked neighbours, C and I, are both blue



--	--	--	--	--	--



www.au.edu.pk

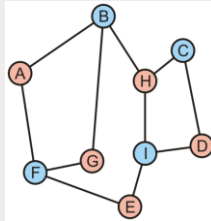
Air University, Islamabad

www.imranhsan.com



BIPARTITE GRAPHS

The queue is empty, the graph is bipartite



www.au.edu.pk

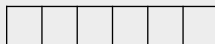
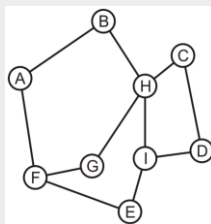
Air University, Islamabad

www.imranhsan.com



BIPARTITE GRAPHS

Consider the other graph which was claimed to be not bipartite



www.au.edu.pk

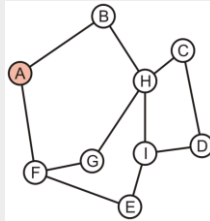
Air University, Islamabad

www.imranhsan.com



BIPARTITE GRAPHS

Push A onto the queue and colour it red



www.au.edu.pk

Air University, Islamabad

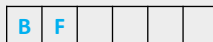
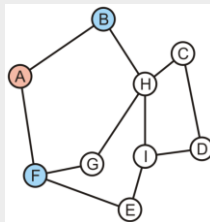
www.imranhsan.com



BIPARTITE GRAPHS

Pop A off the queue:

- Its neighbours are unmarked: colour them blue and push them onto the queue



www.au.edu.pk

Air University, Islamabad

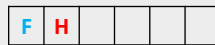
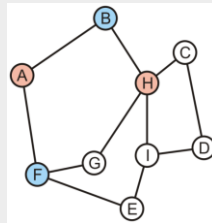
www.imranhsan.com



BIPARTITE GRAPHS

Pop B off the queue:

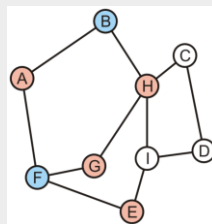
- Its one neighbour, A, is red
- The other neighbour, H, is unmarked: colour it red and push it onto the queue



BIPARTITE GRAPHS

Pop F off the queue:

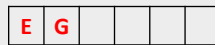
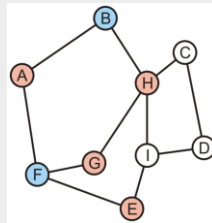
- Its one neighbour, A, is red
- The other neighbours, E and G, are unmarked: colour them red and push it onto the queue



BIPARTITE GRAPHS

Pop H off the queue—it is red:

- Its one neighbour, G, is already red
- The graph is not bipartite



- Definition
 - Cycles that contains either an even number or an odd number of vertices are said to be *even cycles* and *odd cycles*, respectively
- Theorem
 - A graph is bipartite if and only if it does not contain any odd cycles

