DATA STRUCTURES AND ALGORITHMS

LECTURE 22
CONNECTEDNESS

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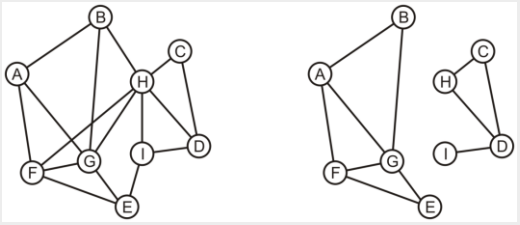
CONNECTED

- First, let us determine whether one vertex is connected to another
  - v_j is connected to v_k if there is a path from the first to the second
- Strategy:
  - Perform a breadth-first traversal starting at v_j
  - While looping, if the vertex v_k ever found to be adjacent to the front of the queue, return true
  - If the loop ends, return false
- Consider implementing a breadth-first traversal on a graph:
  - Choose any vertex, mark it as visited and push it onto queue
  - While the queue is not empty:
    - Pop to top vertex v from the queue
    - For each vertex adjacent to v that has not been visited:
      - Mark it visited, and
      - Push it onto the queue
- This continues until the queue is empty
  - Note: if there are no unvisited vertices, the graph is connected,

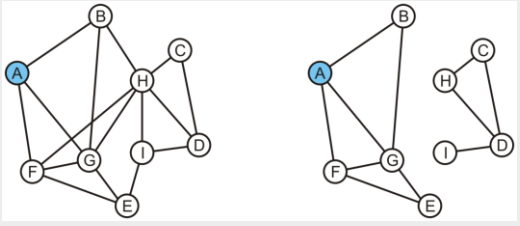
DETERMINING CONNECTIONS

Is A connected to D?



DETERMINING CONNECTIONS

Vertex A is marked as visited and pushed onto the queue

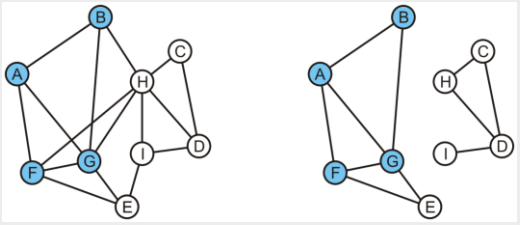


A

A

DETERMINING CONNECTIONS

Pop the head, A, and mark and push B, F and G



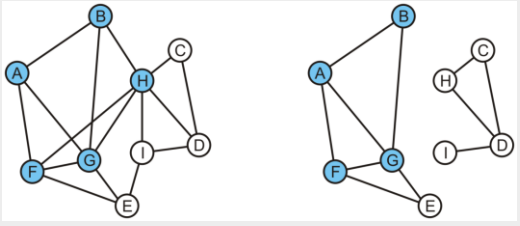
B	F	G			

B	F	G			

DETERMINING CONNECTIONS

Pop B and mark and, in the left graph, mark and push H

- On the right graph, B has no unvisited adjacent vertices

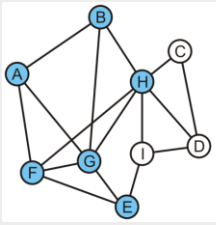


F	G	H			

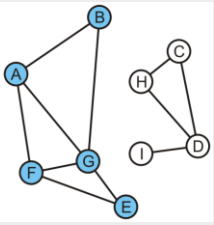
F	G				

DETERMINING CONNECTIONS

Popping F results in the pushing of E



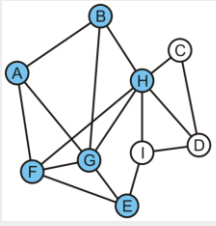
G	H	E			



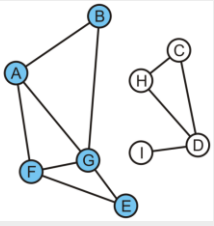
G	E				

DETERMINING CONNECTIONS

In either graph, G has no adjacent vertices that are unvisited



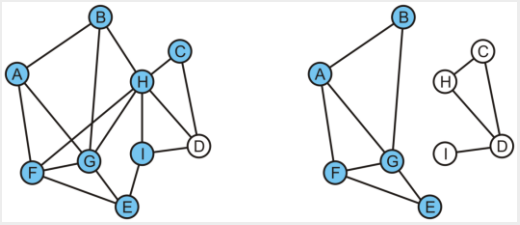
H	E				



E					

DETERMINING CONNECTIONS

Popping H on the left graph results in C and I being pushed

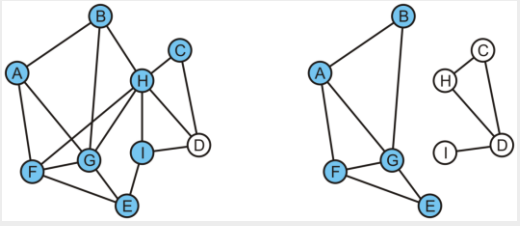


C	I	D			

DETERMINING CONNECTIONS

The queue op the right is empty

- We determine A is not connected to D

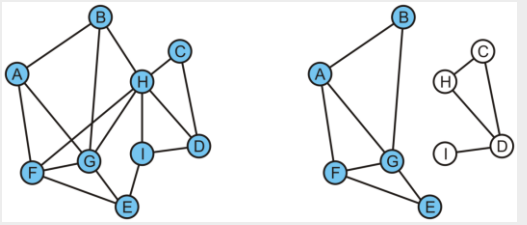


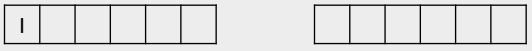
C	I	D			

DETERMINING CONNECTIONS

On the left, we pop C and return `true` because D is adjacent to C

- In the left graph, A is connected to D

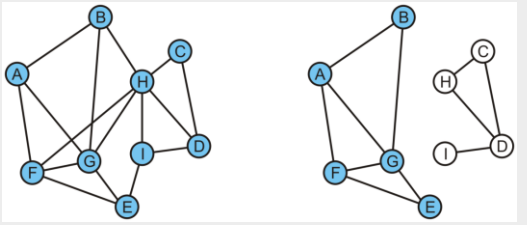


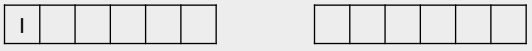


DETERMINING CONNECTIONS

On the left, we pop C and return `true` because D is adjacent to C

- In the left graph, A is connected to D





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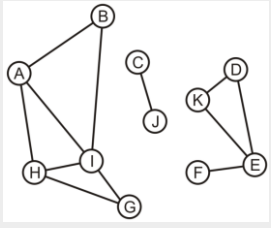
CONNECTED COMPONENTS

- If we continued the traversal, we would find all vertices that are connected to A
- Suppose we want to partition the vertices into connected sub-graphs
  - While there are unvisited vertices in the tree:
    - Select an unvisited vertex and perform a traversal on that vertex
    - Each vertex that is visited in that traversal is added to the set initially containing the initial unvisited vertex
  - Continue until all vertices are visited
- We would use a disjoint set data structure for maximum efficiency

CONNECTED COMPONENTS

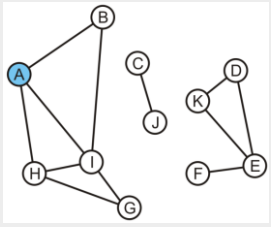
- Here we start with a set of singletons



A	B	C	D	E	F	G	H	I	J	K
A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**

CONNECTED COMPONENTS

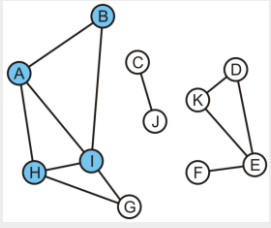
The vertex A is unvisited, so we start with it



A	B	C	D	E	F	G	H	I	J	K
A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**

CONNECTED COMPONENTS

Take the union of with its adjacent vertices: {A, B, H, I}

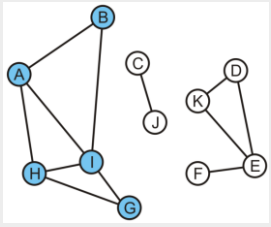


A	B	C	D	E	F	G	H	I	J	K
A	**A**	**C**	**D**	**E**	**F**	**G**	**A**	**A**	**J**	**K**

CONNECTED COMPONENTS

As the traversal continues, we take the union of the set {G} with the set containing H: {A, B, G, H, I}

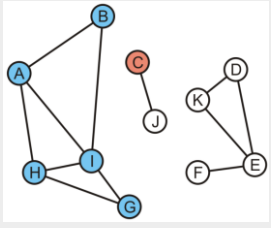
- The traversal is finished



A	B	C	D	E	F	G	H	I	J	K
A	A	C	D	E	F	A	A	A	J	K

CONNECTED COMPONENTS

Start another traversal with C: this defines a new set {C}

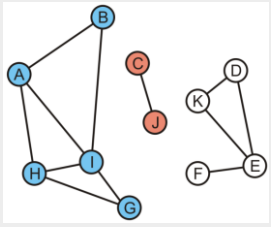


A	B	C	D	E	F	G	H	I	J	K
A	A	C	D	E	F	A	A	A	J	K

CONNECTED COMPONENTS

We take the union of {C} and its adjacent vertex J: {C, J}

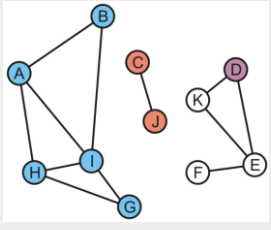
- This traversal is finished



A	B	C	D	E	F	G	H	I	J	K
A	A	C	D	E	F	A	A	A	C	K

CONNECTED COMPONENTS

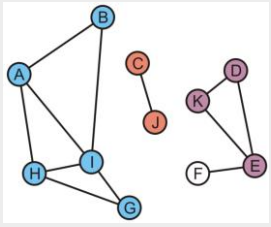
We start again with the set {D}



A	B	C	D	E	F	G	H	I	J	K
A	A	C	D	E	F	A	A	A	C	K

CONNECTED COMPONENTS

K and E are adjacent to D, so take the unions creating {D, E, K}

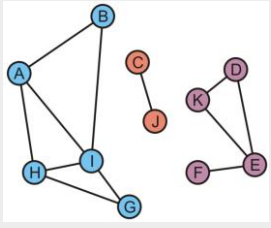


A	B	C	D	E	F	G	H	I	J	K
A	A	C	D	D	F	A	A	A	C	D

CONNECTED COMPONENTS

Finally, during this last traversal we find that F is adjacent to E

- Take the union of {F} with the set containing E: {D, E, F, K}

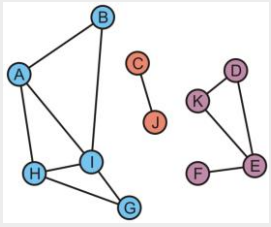


A	B	C	D	E	F	G	H	I	J	K
A	A	C	D	D	D	A	A	A	C	D

CONNECTED COMPONENTS

All vertices are visited, so we are done

- There are three connected sub-graphs {A, B, G, H, I}, {C, J}, {D, E, F, K}



A	B	C	D	E	F	G	H	I	J	K
A	A	C	D	D	D	A	A	A	C	D