Chapter 09.
Propositional Logic

1. The Foundations of Logic
2. Short Recapitulation of Propositional Logic

Knowledge Representation & Reasoning

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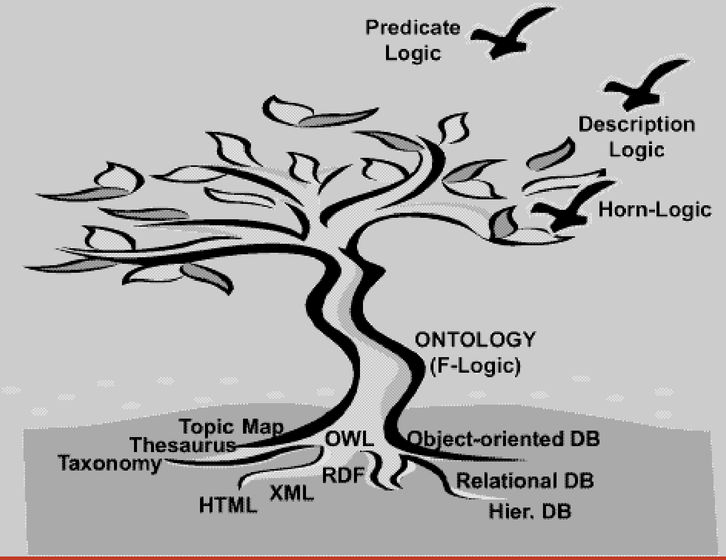

The Foundation of Logic

Ontologies and Logic

An ontology is an explicit, formal specification of a shared conceptualization.

acc. to Thomas R. Gruber: A Translation Approach to Portable Ontology Specifications. Knowledge Acquisition, 5(2):199-220, 1993.

↓

formal (machine readable) semantics

↓

mathematical logic

The Foundations of Logic

Definition (for our lecture):

Logic is the study of how to make formal correct deductions and inferences.

Why "formal logic"? ⇒ to enable automation

"The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate, without further ado, to see who is right."

Leibniz in a letter to Ph.J. Spener,

July 1687

The Foundations of Logic

Syntax: symbols without meaning

defines rules, how to construct well-formed and valid sequences of symbols (strings)

Semantic: meaning of syntax

defines rules how the meaning of complex sequences of symbols can be derived from atomic sequences of symbols.

Syntax

If (i<0) then display ("negative account!")

assignment of meaning

print the message "negative account!", if the account balance is negative

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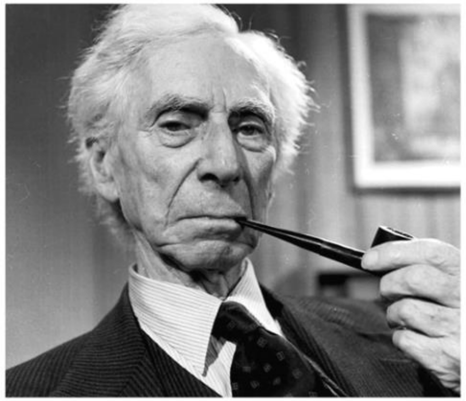
The Importance of Semantics

Bertrand Russell
(1872 - 1970)



Why should I care about semantics?

Well, from a philosophical POV, we need to specify the relationship between statements in the logic and the existential phenomena they describe.

From a practical POV, in order to specify, build and test (ontology-based) tools/systems we need to precisely define relationships (like entailment) between logical statements – this defines the intended behavior of tools/systems.

Variants of Semantics

e.g. programming languages

computation of the factorial

Syntax

```
FUNCTION
f(n:natural) :natural;
BEGIN
    IF n=0 THEN f:=1
    ELSE f:=n*f(n-1);
END;
```

Intentional semantics

"**the meaning intended by the user**"
restricts the set of all possible models (meanings) to the meaning intended by the (human) user

Variants of Semantics

e.g. programming languages

Syntax

```
FUNCTION
f(n:natural) :natural;
BEGIN
        IF n=0 THEN f:=1
        ELSE f:=n*f(n-1);
END;
```

computation of the factorial

f: n → n!

Formal semantics

aims to express the meaning of symbol sequences (programs) in a **formal language**, in a way that assertions over the symbol sequences (programs) can be proven by the application of deduction rules.

Variants of Semantics

e.g. programming languages

Syntax

```
FUNCTION
f(n:natural) :natural;
BEGIN
    IF n=0 THEN f:=1
    ELSE f:=n*f(n-1);
END;
```

computation of the factorial

f: n → n!

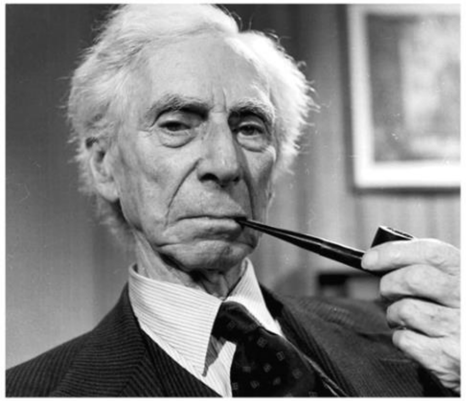
behavior of the program at execution

Procedural semantics

the meaning of a language expression (program) is the procedure that takes place internally, whenever the expression does occur.

Semantics and Mathematical Logic

Bertrand Russell
(1872 - 1970)



How do I define the semantics of a mathematical logic?

In mathematical logic we define the semantics in terms of models (a model theory). A model is supposed to be an analogue of (part of) the world being modeled.

Model-theoretic Semantics

Model-theoretic semantics performs the semantic interpretation of artificial and natural languages by "identifying meaning with an exact and formally defined interpretation with a model"

= formal interpretation with a model

e.g. model-theoretic semantics of propositional logic

- Assignment of truth values "true" and "false" to atomic assertions and
- Description of logical connectives with truth tables

Alfred Tarski
(1901-1983)

Model-Theoretic Semantics

Any logic $L := (S, \models)$ consists of

(1) A set of statements S and

(2) An entailment relation $\models$

Let $\Phi \subseteq S$ and $\varphi \in S : \Phi \models \varphi$

φ is a logical consequence of Φ or

from the assertions of Φ follows the assertion of φ

If for 2 assertions $\varphi, \psi \in S$

Both $\{\varphi\} \models \psi$ and $\{\psi\} \models \varphi$

The both assertions φ and ψ are logically equivalent $\varphi \equiv \psi$

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$\forall \quad \exists \quad \vee \quad \wedge \quad \neg \quad \rightarrow \quad \Rightarrow \quad \Leftrightarrow$

Short Recapitulation of Propositional Logic

Propositional Logic – PL

in propositional logic the world consists simply of facts and nothing else (statements of assertions)

Example for propositional logic assertions and deductions:

- If it rains, then road will get wet.
- If the moon is made out of green cheese, then cows can fly.
- If Oliver is in love, then he will be happy.

The world consists out of objects and properties that distinguish one object from another.

Between objects are relations. Some relations are unique, i.e. functions.

Propositional Logic – PL

Syntax:

- Logical connectives: Op={ ¬,∧,∨,→,↔,(,) },
- a set of symbols Σ
- with Σ ∩ Op = ∅ and {true, false}

connective	name	Intentional meaning
¬	negation	"not"
∧	conjunction	"and"
∨	disjunction	"or"
→	implication	"if – then"
↔	equivalence	"if, and only if, then"

- Production rules for propositional formulae (propositions):
  - all atomic formulas are propositions (all elements of Σ)
  - if φ is a proposition, then also ¬φ
  - if φ and ψ are propositions, then also φ∧ψ, φ∨ψ, φ→ψ, φ ↔ ψ

- Priority: ¬ prior to ∧, ∨ prior to →, ↔

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Propositional Logic – How to model facts?

Simple Assertion	Modeling
The Moon is made of green cheese	g
It rains	r
The street is getting wet	n

Composed Assertion	Modeling
if it rains, then the street will get wet.	$r \rightarrow n$
If it rains and the street does not get wet, then the moon is made of green cheese.	$(r \land \neg n) \rightarrow g$

Propositional Logic – Model-theoretic Semantics

Interpretation I:

Mapping of all atomic propositions to {t,f}.

If F is a formula and I an Interpretation, then I(F) is a truth value computed from F and I via truth tables.

I(p)	I(q)	I(¬p)	I(p∨q)	I(p∧q)	I(p→q)	I(p↔q)
F	F	T	F	F	T	T
F	T	T	T	F	T	F
T	F	F	T	F	F	F
T	T	F	T	T	T	T

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Propositional Logic – Model-theoretic Semantics

We write I ⊨ F, if I(F)=t,

and call Interpretation I, a Model of formula F.

Rules of Semantics:

- I is model of ¬φ, iff I is not a model of φ
- I is model of (φ∧ψ), iff I is a model of φ AND of ψ
- ...

Basic concepts:

- tautology
- satisfiable
- refutable
- unsatisfiable (contradiction)

Some Properties of Propositional Logic – PL

Decidability

All true entailments can be found, and all false entailments can be refuted, if you spent enough time.

⇒ there always exist terminating automatic theorem proofer

Another useful property: **semantic entailment/inference**

$\{\varphi_1,\ldots,\varphi_n\} \vDash \varphi$ holds, iff

$(\varphi_1 \land \ldots \land \varphi_n) \rightarrow \varphi$ is a tautology

syntactic entailment/inference

The decision, if an assertion is a tautology, can be made via truth tables

In principle this equals the evaluation of all possible interpretations