Design and Analysis of Algorithms

06-04
Minimum Spanning Tree

Kruskal's Algorithm and Prim's Algorithm

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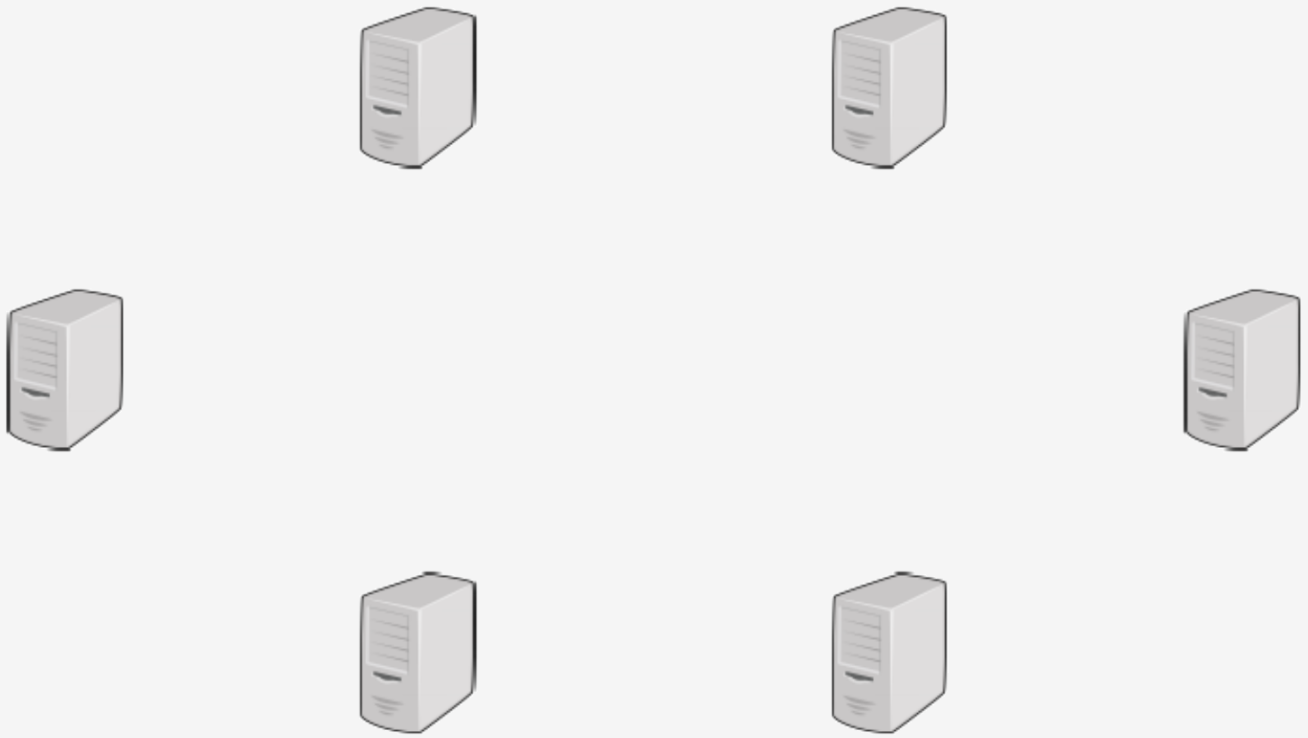
Connecting Computers by Wires



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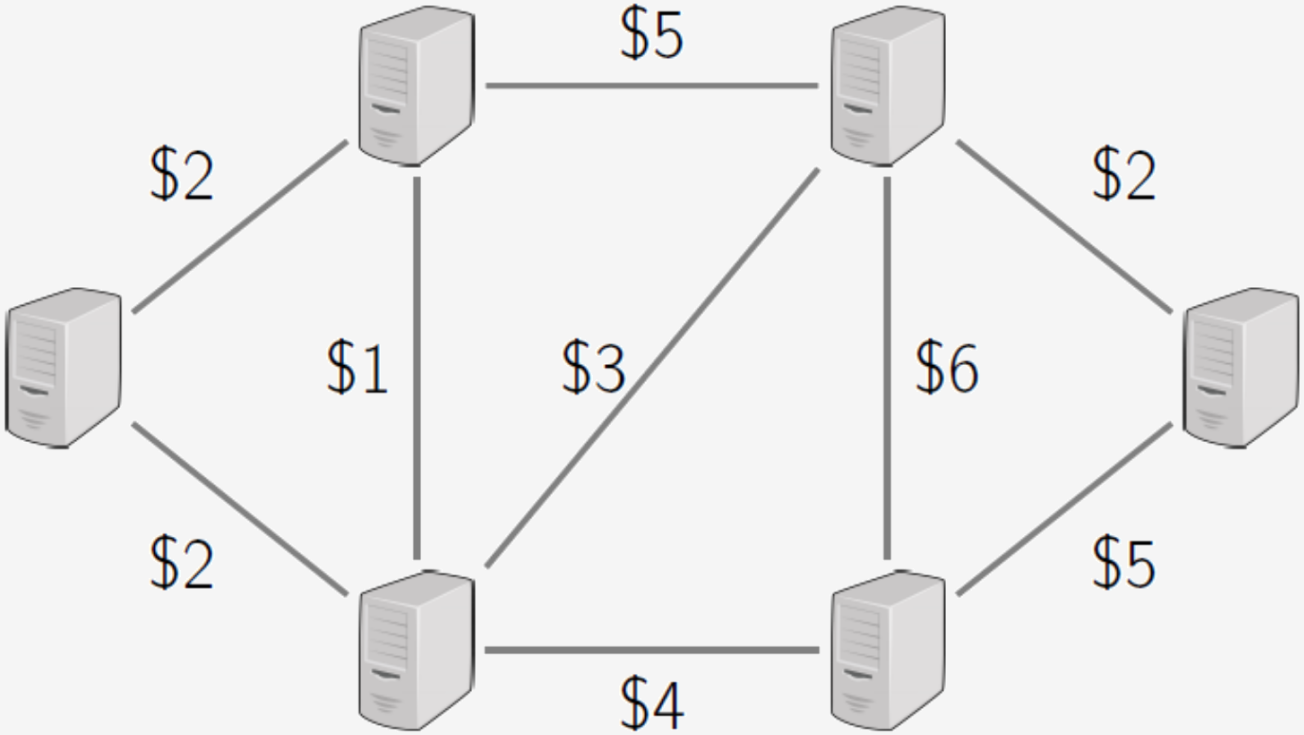

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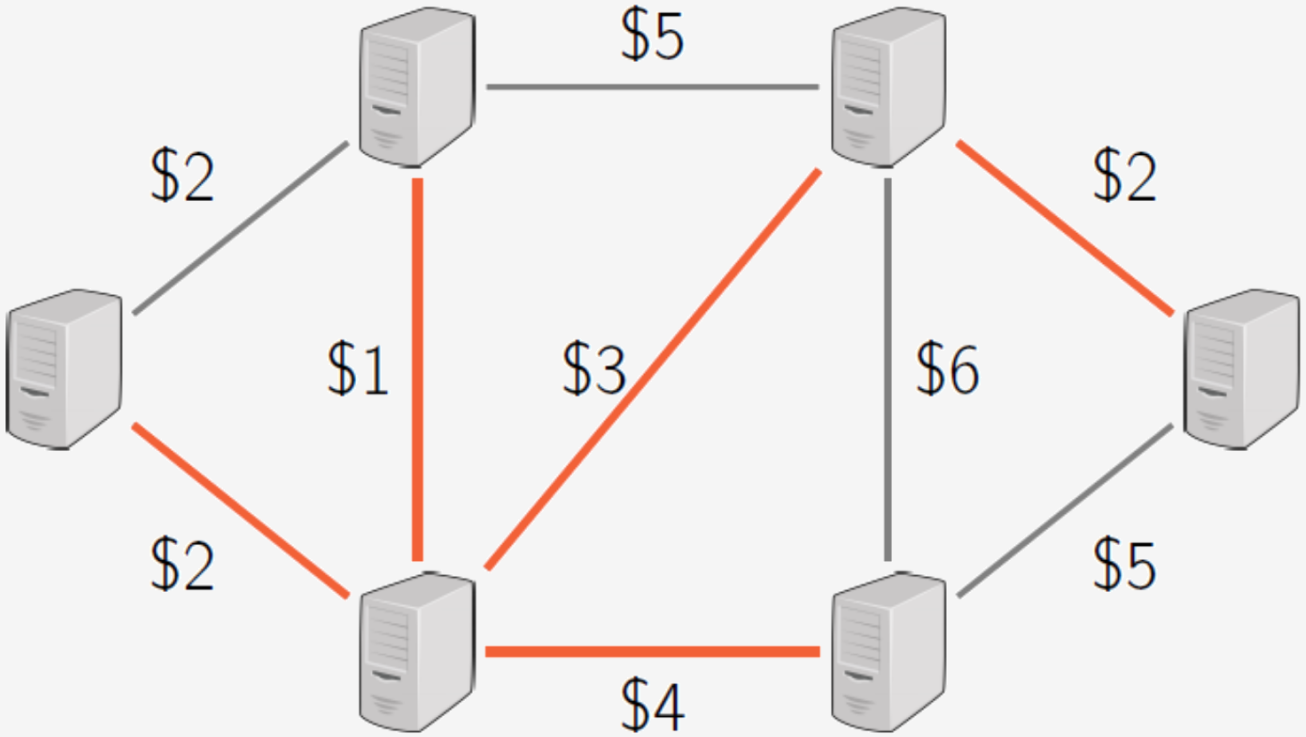

Building Roads

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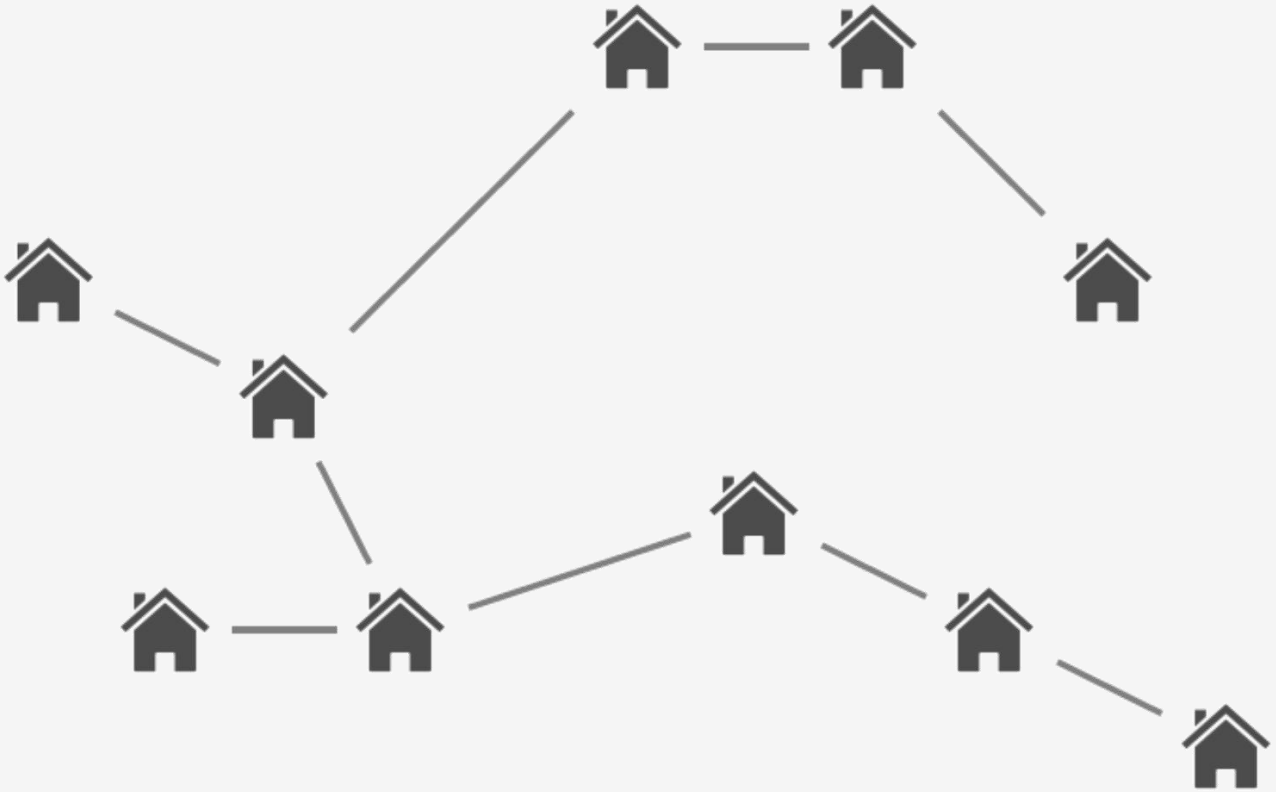


Minimum spanning tree (MST)

Input:

A connected, undirected graph G = (V , E) with positive edge weights.

Output:

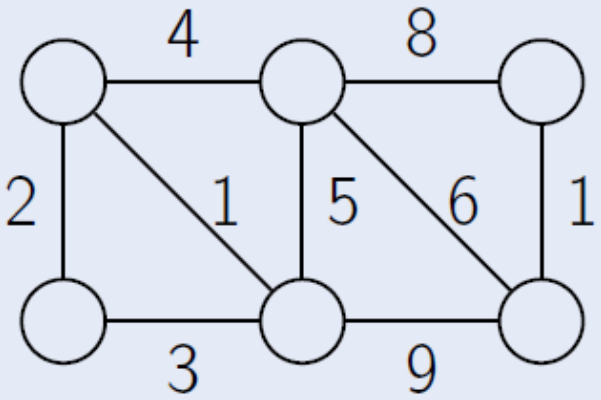
A subset of edges E’ ⊆ E of minimum total weight such that the graph (V , E’) is connected.

Remark

The set E’ always forms a tree.

Kruskal's Algorithm

repeatedly add the next lightest edge if this doesn't produce a cycle



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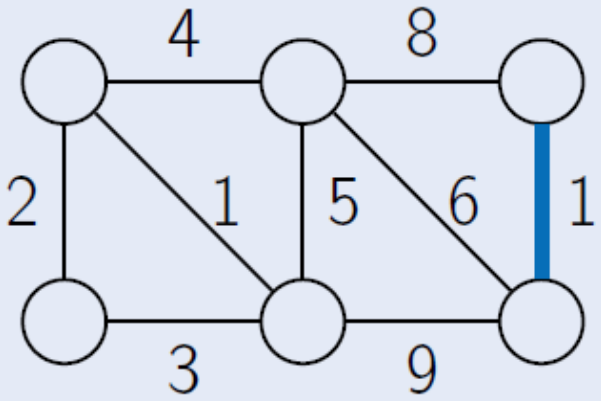

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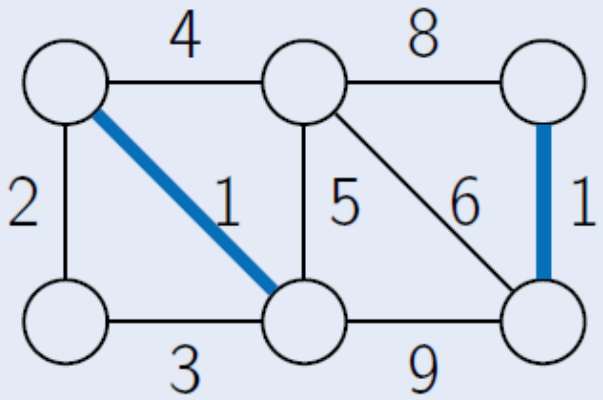

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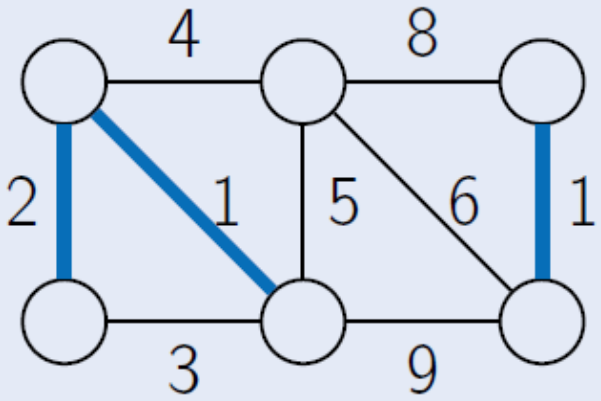

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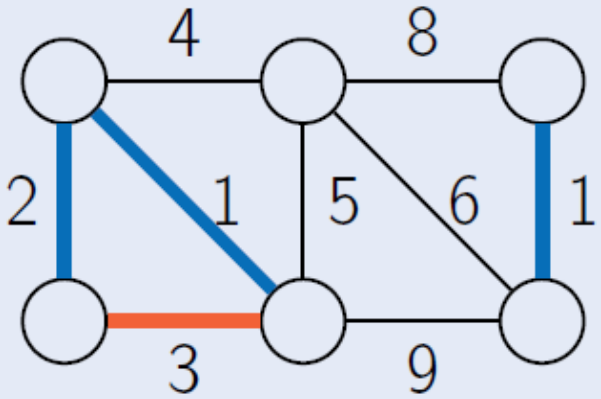

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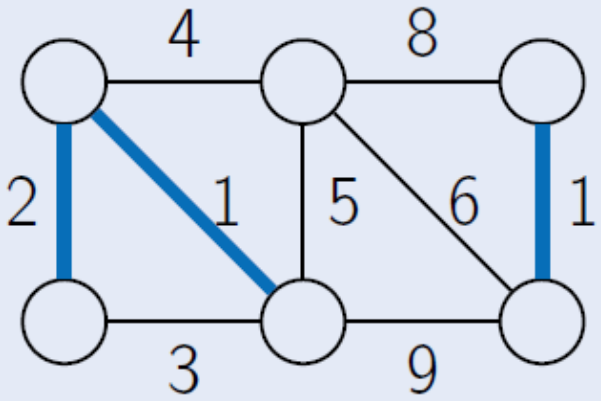

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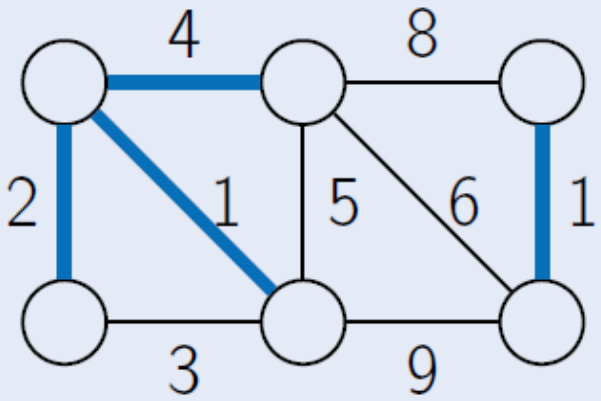

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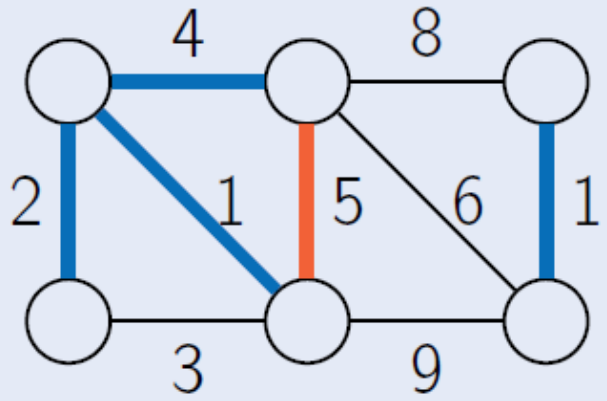

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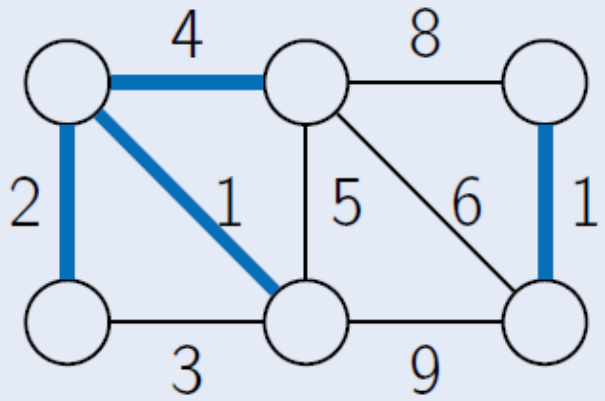

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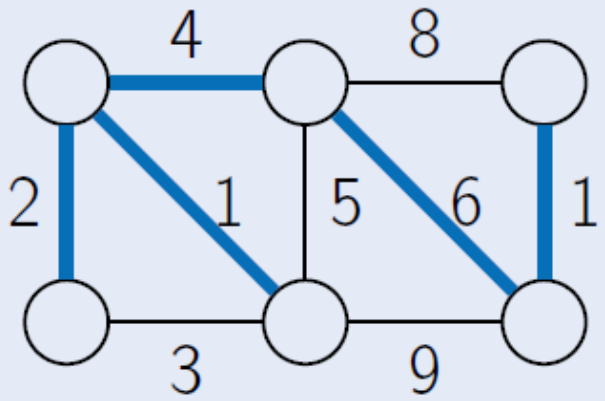

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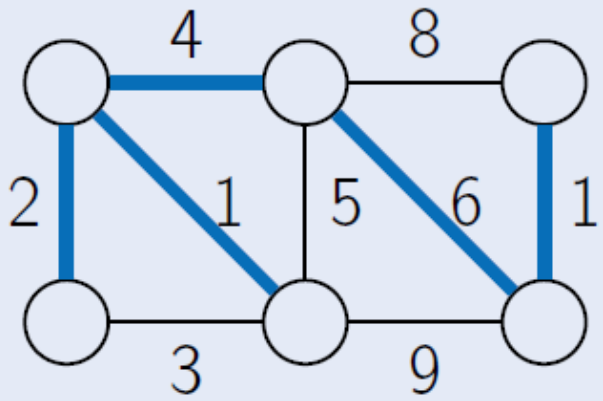

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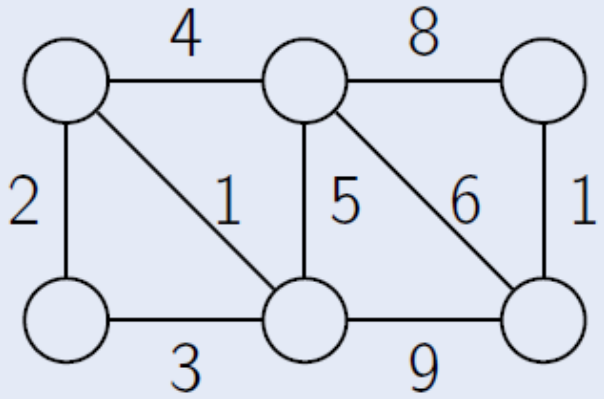

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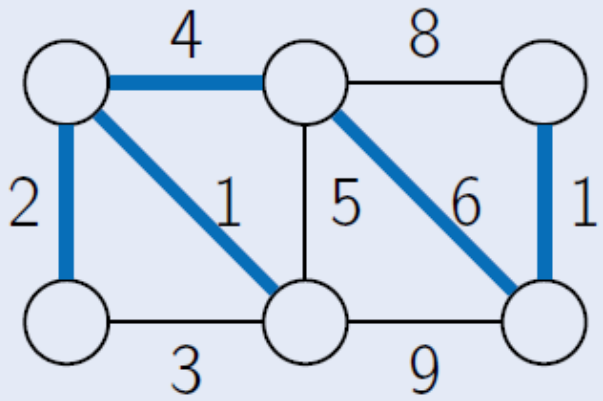

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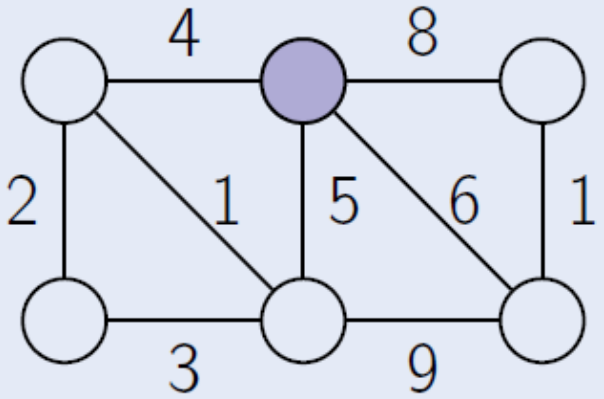

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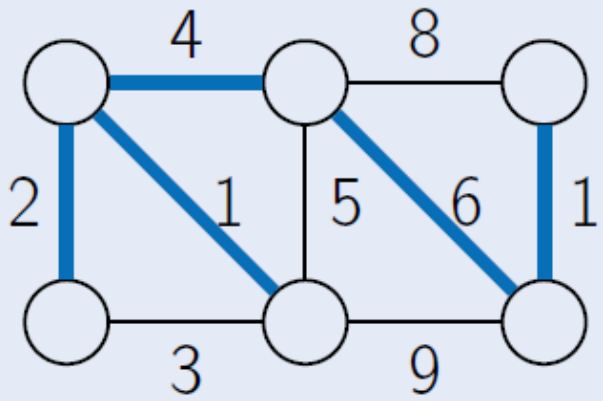

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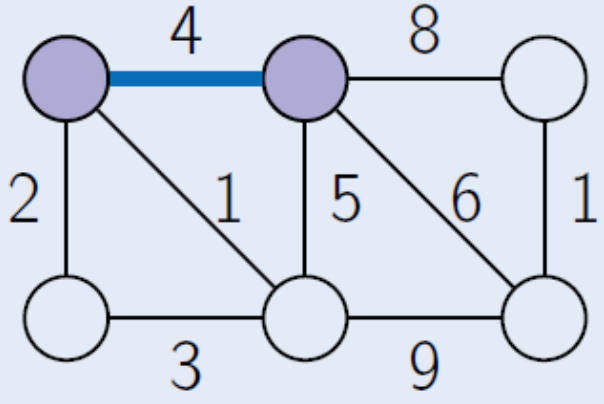

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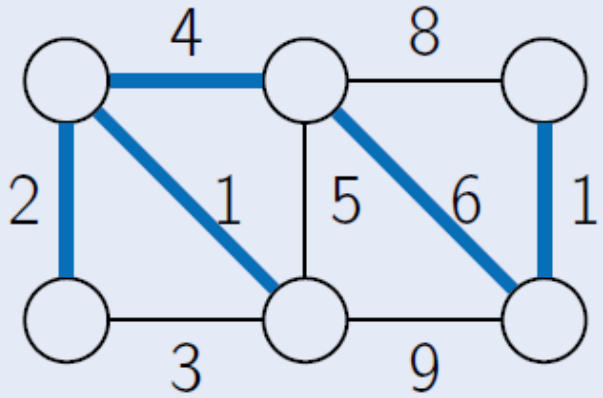

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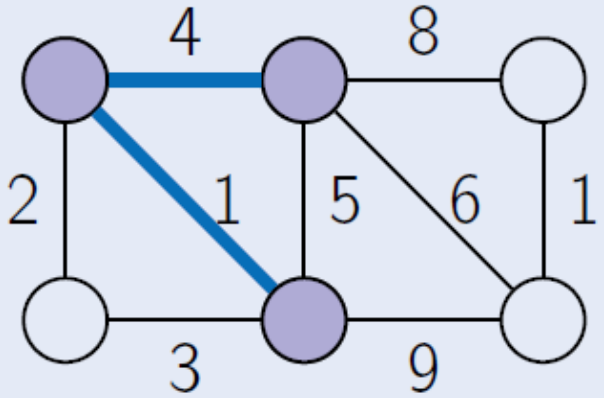

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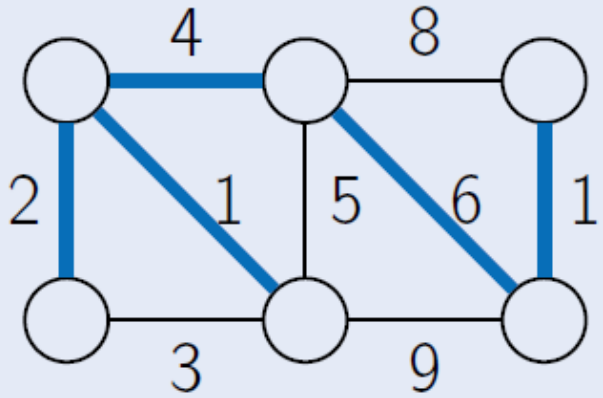

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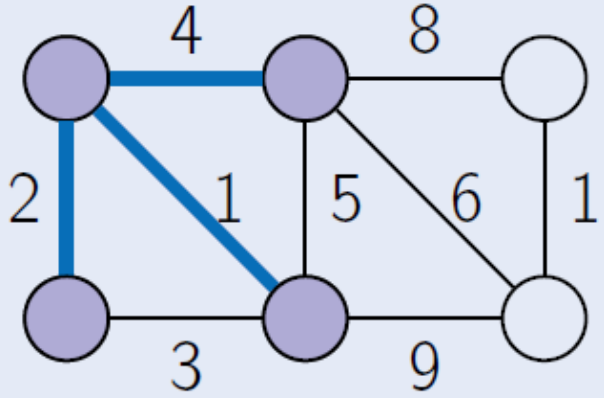

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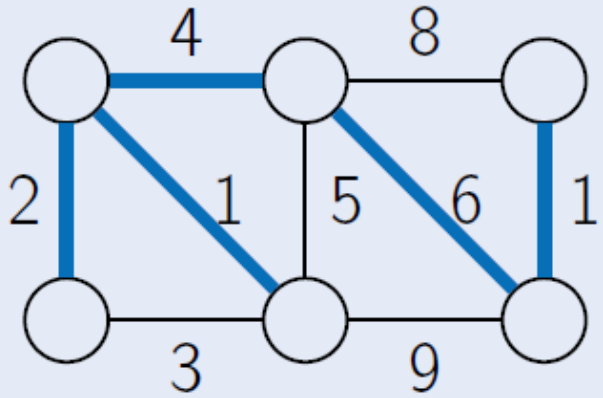

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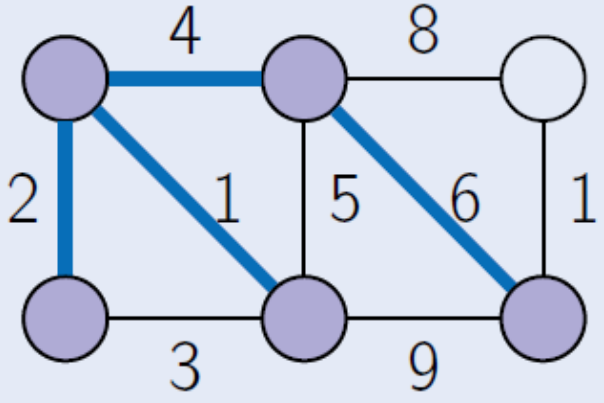

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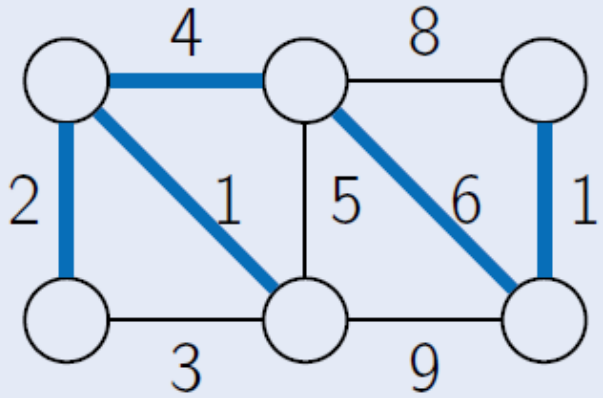

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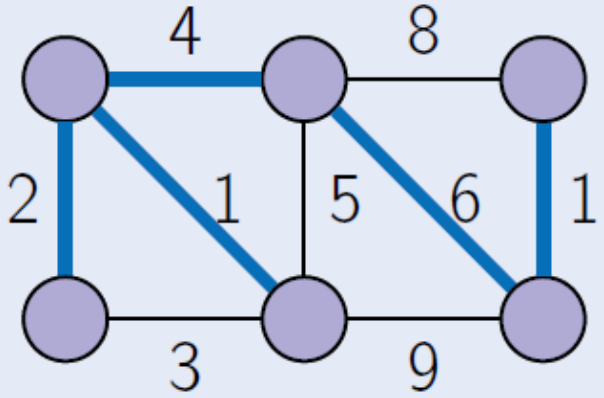

Kruskal's Algorithm

Algorithm:

Repeatedly add to X the next lightest edge e that doesn't produce a cycle

At any point of time, the set X is a forest, that is, a collection of trees

Use disjoint sets data structure

Initially, each vertex lies in a separate set each set is the set of vertices of a connected component

To check whether the current edge {u, v} produces a cycle, we check whether u and v belong to the same set

Kruskal's Algorithm

```
Kruskal(G)
   for all u ∈ V :
       MakeSet(v)
   X ← empty set
   sort the edges E by weight
   for all {u, v} ∈ E in non-decreasing weight order:
       if Find(u) ≠ Find(v):
           add {u, v} to X
           Union(u, v)
   return X.
```

Prim's Algorithm

X is always a subtree, grows by one edge at each iteration

We add a lightest edge between a vertex of the tree and a vertex not in the tree

Very similar to Dijkstra's algorithm

Prim's Algorithm

```
Prim(G)
    for all u ∈ V:
        cost[u] ← ∞, parent[u] ← nil
    pick any initial vertex u0
    cost[u0] ← 0
    PrioQ ← MakeQueue(V) {priority is cost}
    while PrioQ is not empty:
        v ← ExtractMin(PrioQ)
        for all {v, z} ∈ E:
            If z ∈ PrioQ and cost[z] > w(v, z):
                cost[z] ← w(v, z), parent[z] ← v
                ChangePriority(PrioQ, z, cost[z]).
```

Summary

Kruskal:

Repeatedly add the next lightest edge if this doesn't produce a cycle; use disjoint sets to check whether the current edge joins two vertices from different components.

Prim:

Repeatedly attach a new vertex to the current tree by a lightest edge; use priority queue to quickly find the next lightest edge.