

Design and Analysis of Algorithms

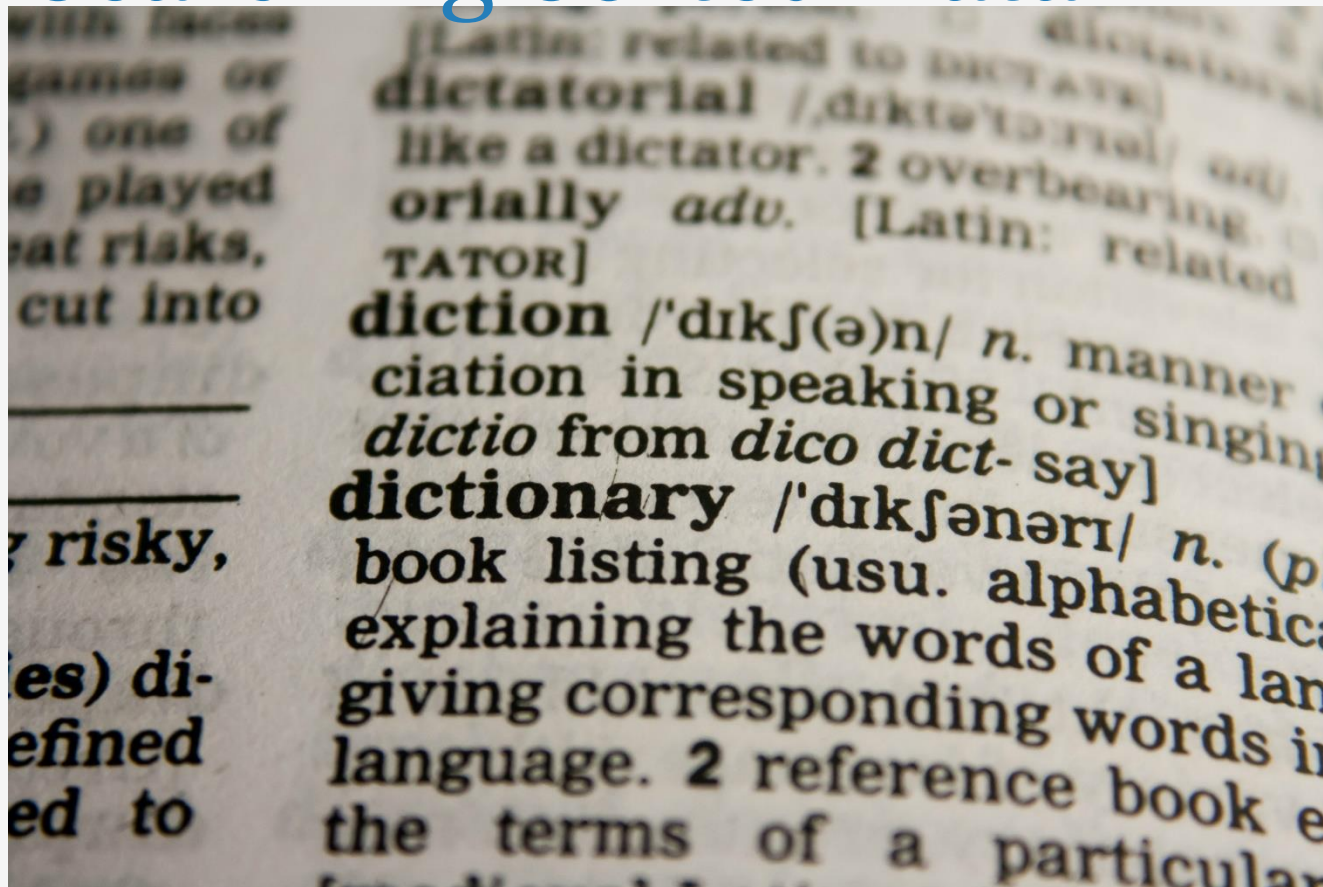
03-03 Divide – and – Conquer

Binary Search

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Searching Sorted Data



Searching in a Sorted Array

Input: A sorted array $A[\text{low} \dots \text{high}]$
 $(\forall \text{low} \leq i < \text{high} : A[i] \leq A[i + 1])$.
 A key k .

Output: An index, i , $(\text{low} \leq i \leq \text{high})$ where $A[i] = k$.
 Otherwise, the greatest index i , where $A[i] < k$.
 Otherwise $(k < A[\text{low}])$, the result is $\text{low} - 1$.

Searching in a Sorted Array

3	5	8	20	20	50	60
1	2	3	4	5	6	7

Searching in a Sorted Array

Search(2) → 0

3	5	8	20	20	50	60
1	2	3	4	5	6	7

Searching in a Sorted Array

Search(2) → 0

Search(3) → 1

3	5	8	20	20	50	60
1	2	3	4	5	6	7

Searching in a Sorted Array

Search(2) → 0

Search(3) → 1

Search(4) → 1

3	5	8	20	20	50	60
1	2	3	4	5	6	7

Searching in a Sorted Array

Search(2) → 0

Search(3) → 1

Search(4) → 1

Search(20) → 4

3	5	8	20	20	50	60
1	2	3	4	5	6	7

Searching in a Sorted Array

Search(2) → 0

Search(3) → 1

Search(4) → 1

Search(20) → 4

Search(20) → 5

3	5	8	20	20	50	60
1	2	3	4	5	6	7

Searching in a Sorted Array

Search(2) → 0

Search(3) → 1

Search(4) → 1

Search(20) → 4

Search(20) → 5

Search(60) → 7

3	5	8	20	20	50	60
1	2	3	4	5	6	7

Searching in a Sorted Array

Search(2) → 0

Search(3) → 1

Search(4) → 1

Search(20) → 4

Search(20) → 5

Search(60) → 7

Search(90) → 7

3	5	8	20	20	50	60
1	2	3	4	5	6	7

Recursive Solution

```
BinarySearch(A, low, high, key)
```

```
  if high < low:
```

```
    return low - 1
```

```
  mid ← [ low + (high-low)/2 ]
```

```
  if key = A[mid]:
```

```
    return mid
```

```
  else if key < A[mid]:
```

```
    return BinarySearch(A, low, mid - 1, key)
```

```
  else:
```

```
    return BinarySearch(A, mid + 1, high, key)
```

Example: Searching for the key 50

Search(2) → 0

Search(3) → 1

Search(4) → 1

Search(20) → 4

Search(20) → 5

Search(60) → 7

Search(90) → 7

3	5	8	10	12	15	18	20	20	50	60
1	2	3	4	5	6	7	8	9	10	11

Example: Searching for the key 50

BinarySearch(A, 1, 11, 50)

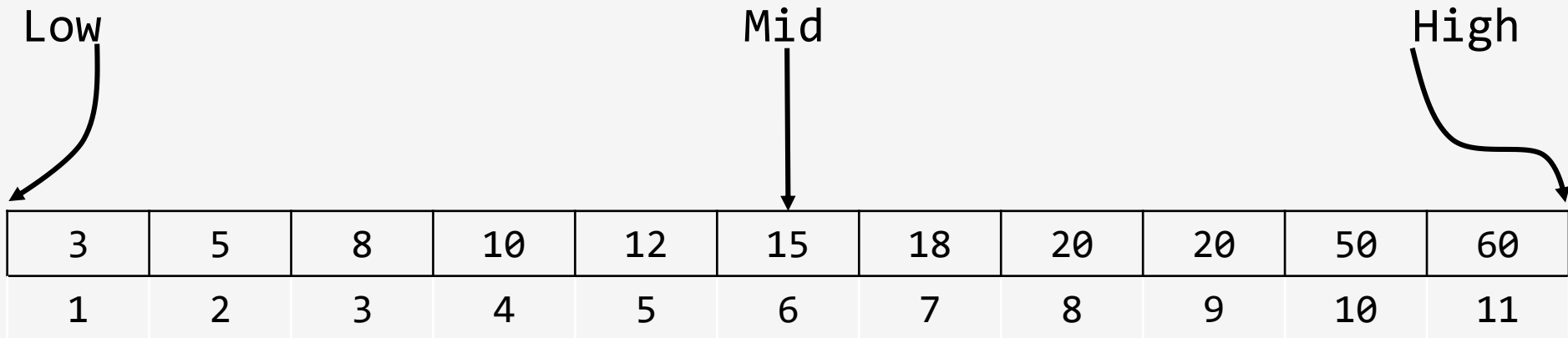
3	5	8	10	12	15	18	20	20	50	60
1	2	3	4	5	6	7	8	9	10	11

Low

High

Example: Searching for the key 50

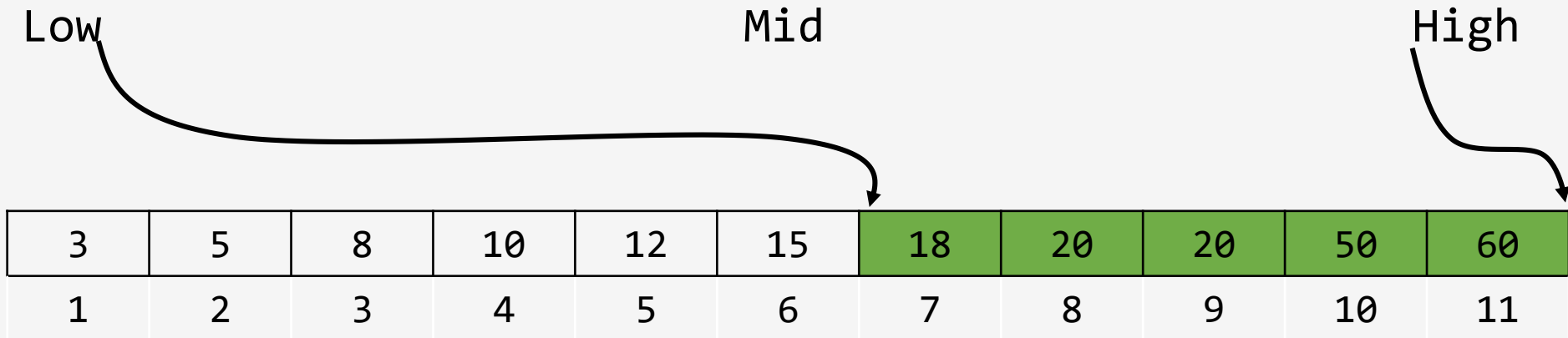
BinarySearch(A, 1, 11, 50)



Example: Searching for the key 50

BinarySearch(A, 1, 11, 50)

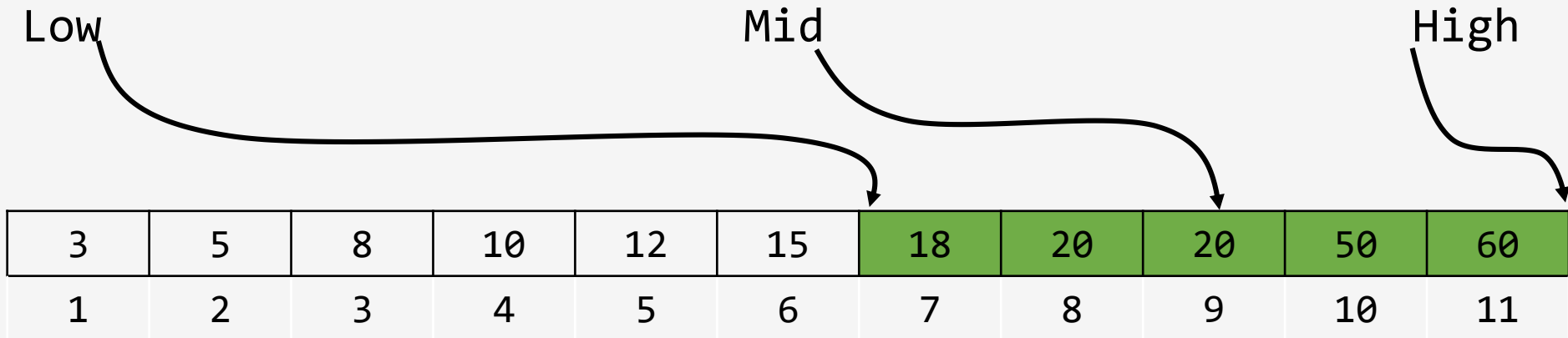
BinarySearch(A, 7, 11, 50)



Example: Searching for the key 50

BinarySearch(A, 1, 11, 50)

BinarySearch(A, 7, 11, 50)

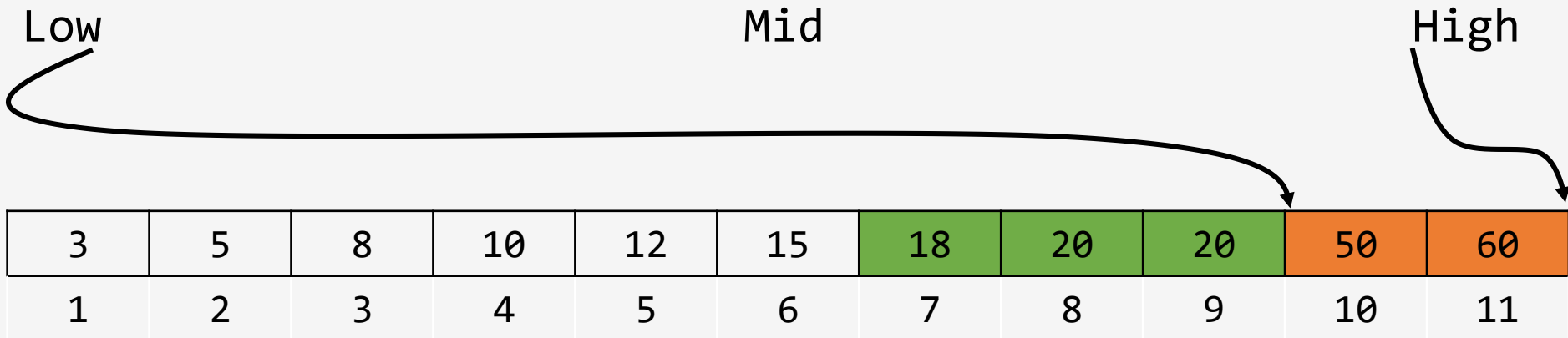


Example: Searching for the key 50

BinarySearch(A, 1, 11, 50)

BinarySearch(A, 7, 11, 50)

BinarySearch(A, 10, 11, 50)

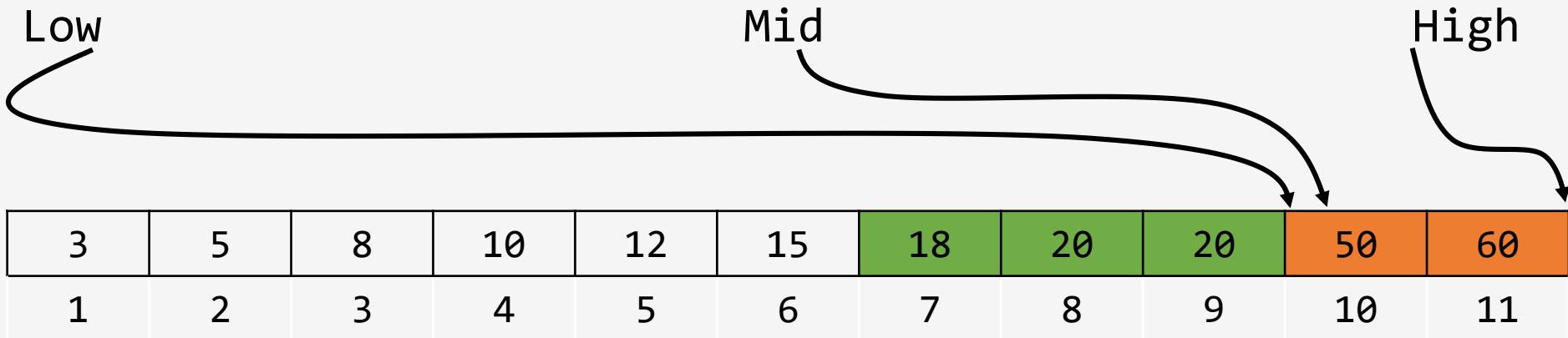


Example: Searching for the key 50

BinarySearch(A, 1, 11, 50)

BinarySearch(A, 7, 11, 50)

BinarySearch(A, 10, 11, 50)

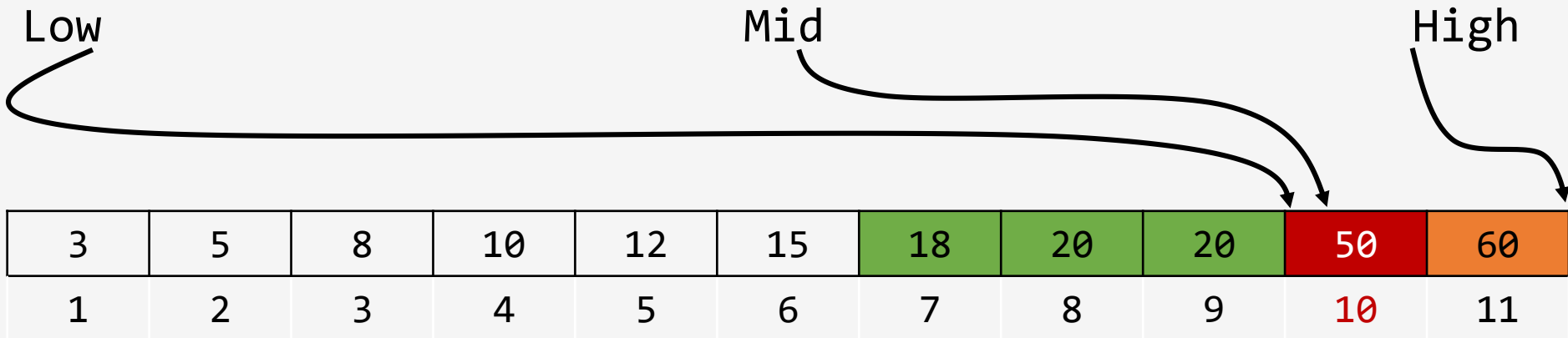


Example: Searching for the key 50

BinarySearch(A, 1, 11, 50)

BinarySearch(A, 7, 11, 50)

BinarySearch(A, 10, 11, 50) → 10



Summary

Break problem into non-overlapping subproblems of the same type.

Recursively solve those subproblems.

Combine results of subproblems.

Recursive Solution

```
BinarySearch(A, low, high, key)
```

```
  if high < low:
```

```
    return low - 1
```

```
  mid ← [ low + (high-low)/2 ]
```

```
  if key = A[mid]:
```

```
    return mid
```

```
  else if key < A[mid]:
```

```
    return BinarySearch(A, low, mid - 1, key)
```

```
  else:
```

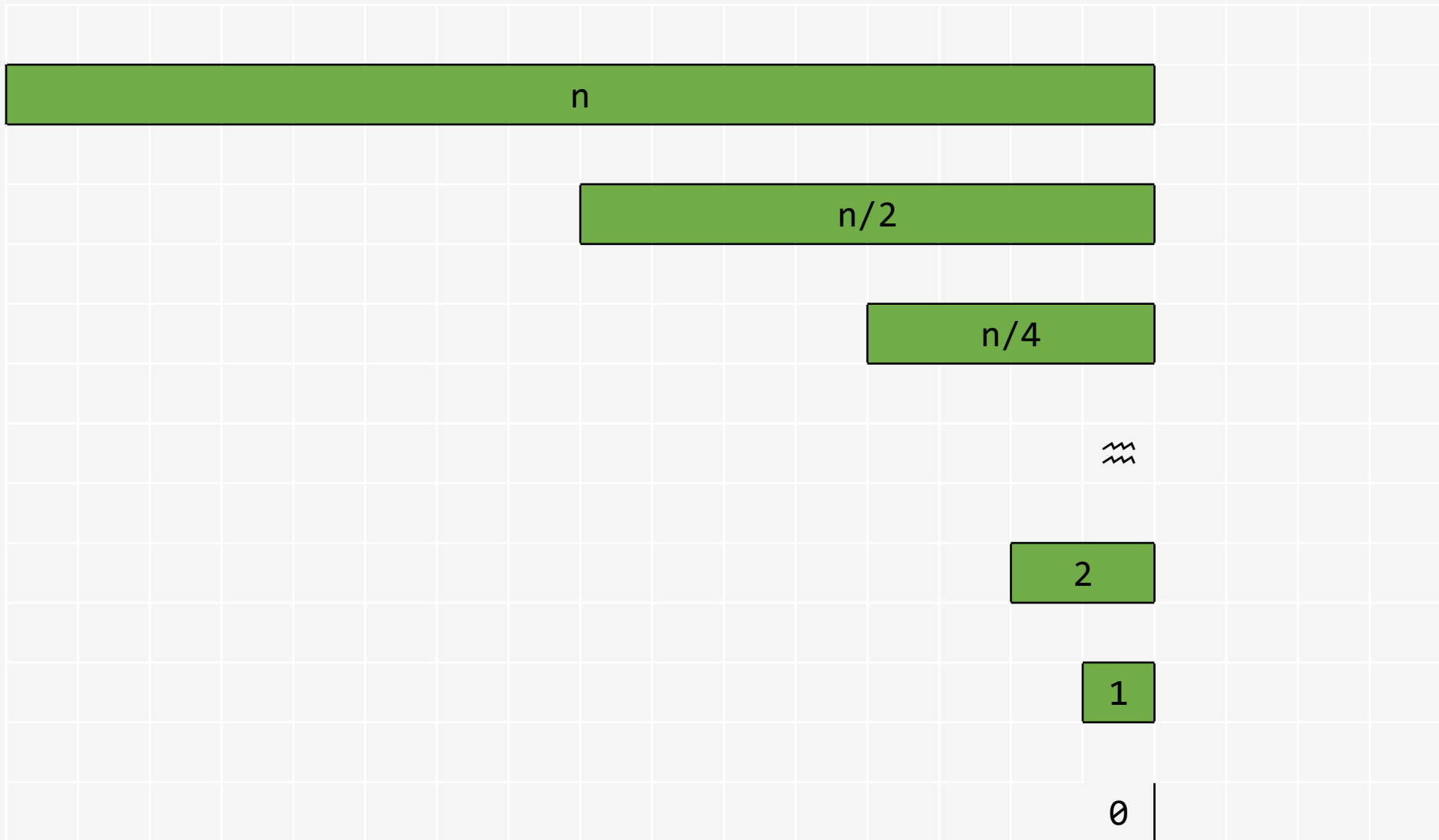
```
    return BinarySearch(A, mid + 1, high, key)
```

Binary Search Recurrence Relation

$$T(n) = T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + c$$

$$T(0) = c$$

Runtime of Binary Search



Runtime of Binary Search

Work

n

C

$n/2$

C

$n/4$

C

⋮

2

C

1

C

\emptyset

C

$$Total = \sum_{i=0}^{\log_2 n} C = \theta(\log_2 n)$$

Iterative Version

```
BinarySearchIt(A, low, high, key)
```

```
while low ≤ high:  
    mid ← ⌊ low + (high-low)/2 ⌋  
    if key = A[mid]:  
        return mid  
    else if key < A[mid]:  
        high = mid - 1  
    else:  
        low = mid + 1  
return low - 1
```

Real Life Example

English	French	Italian	German	Spanish
House	Maison	Casa	Haus	Casa
Chair	Chaise	Sedia	Sessel	Silla
Pimple	Bouton	Foruncolo	Pickel	Espenilla

English (sorted)	French (sorted)	Italian (sorted)	German (sorted)	Spanish (sorted)
Chair	Chaise	Casa	Haus	Casa
House	Bouton	Foruncolo	Pickel	Espenilla
Pimple	Maison	Sedia	Sessel	Silla

Real Life Example

English (sorted)	French (sorted)	Italian (sorted)	German (sorted)	Spanish (sorted)
Chair	Chaise	Casa	Haus	Casa
House	Bouton	Foruncolo	Pickel	Espenilla
Pimple	Maison	Sedia	Sessel	Silla

English
(sorted)

2
1
3

Spanish
(sorted)

1
3
2

Real Life Example

English	French	Italian	German	Spanish
House	Maison	Casa	Haus	Casa
Chair	Chaise	Sedia	Sessel	Silla
Pimple	Bouton	Foruncolo	Pickel	Espenilla

English
(sorted)

2
1
3

Spanish
(sorted)

1
3
2

Real Life Example

